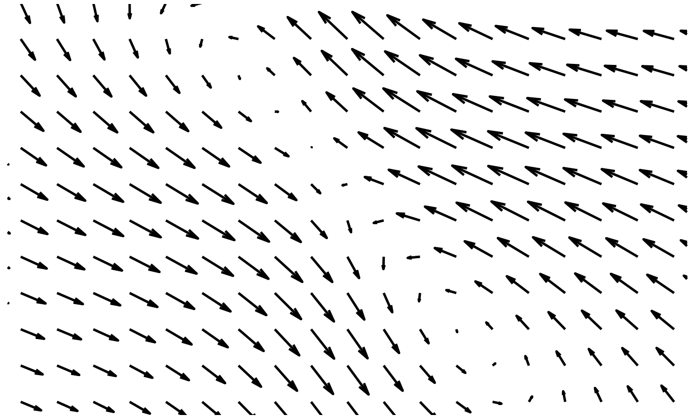
Extreme dissipation at very high Reynolds number



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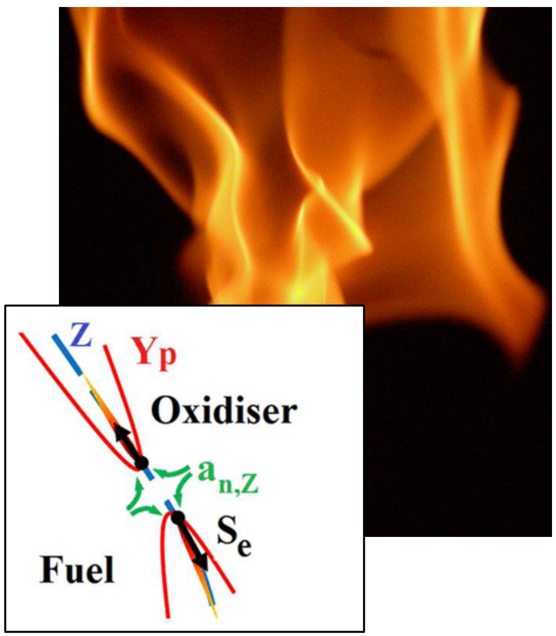
Intense dissipation (or strain-rate) critical in many applications

$$\varepsilon = 2\nu S_{ij}S_{ij}$$

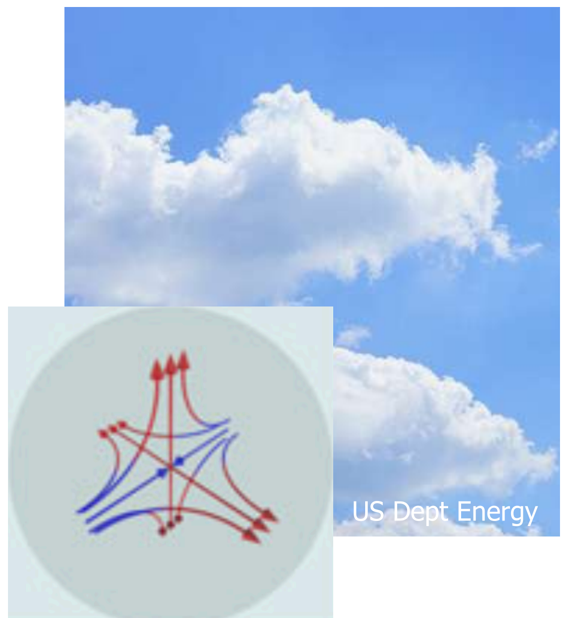
ε – viscous dissipation rate
S_{ij} – strain-rate tensor

local flame extinction



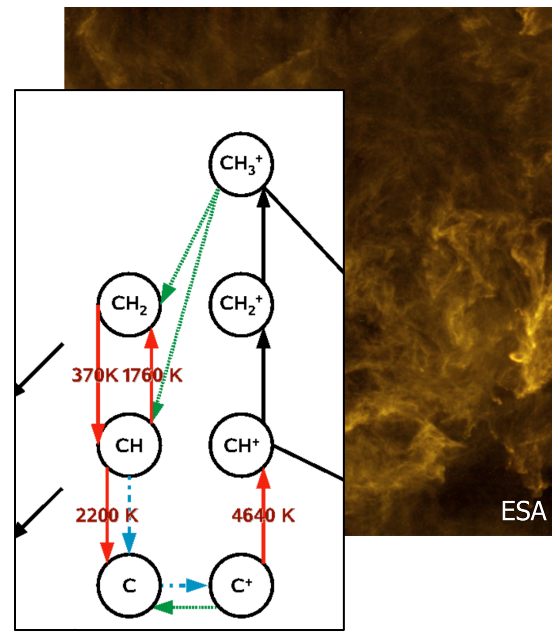
flame front thinning
[Karami et al. 2017 Proc Comb Inst]

droplet collision



~60% collisions in strain regions
[Perrin & Jonker 2016 JFM]
Collision rate depends on dissipation rate
[Franklin et al. 2005 J Atm Sci]

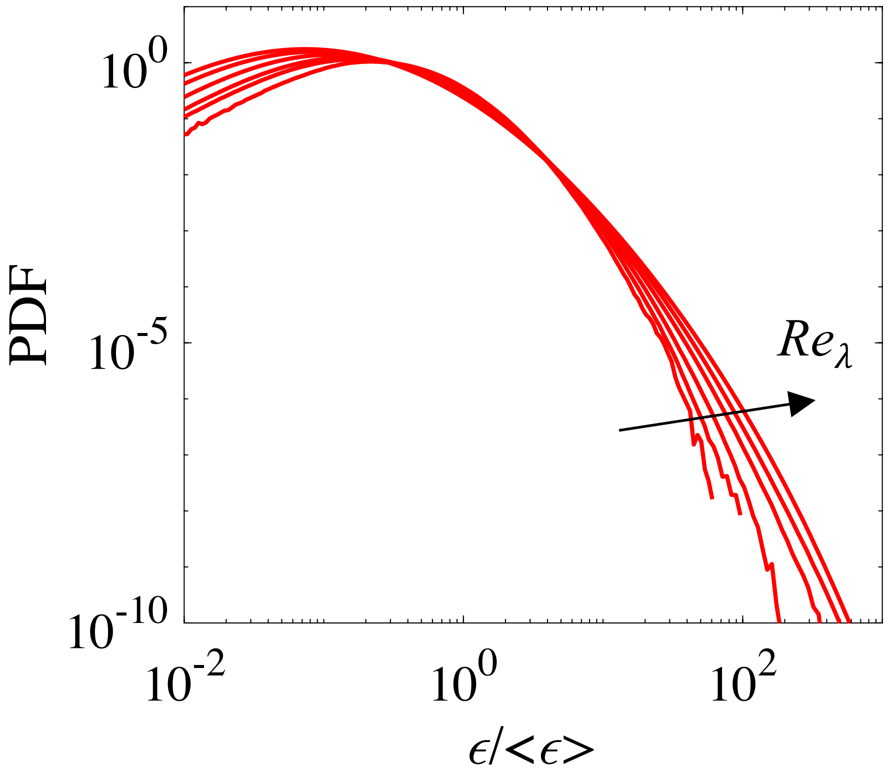
astro-chemistry



heating source
[Godard et al. 2009 A&A]

Extreme dissipation is strongly Reynolds number dependent

- Reliable data available only at low $Re_\lambda \leq 1100$ (DNS)
- Need theories for extrapolation to higher Re_λ (e.g. atmosphere)



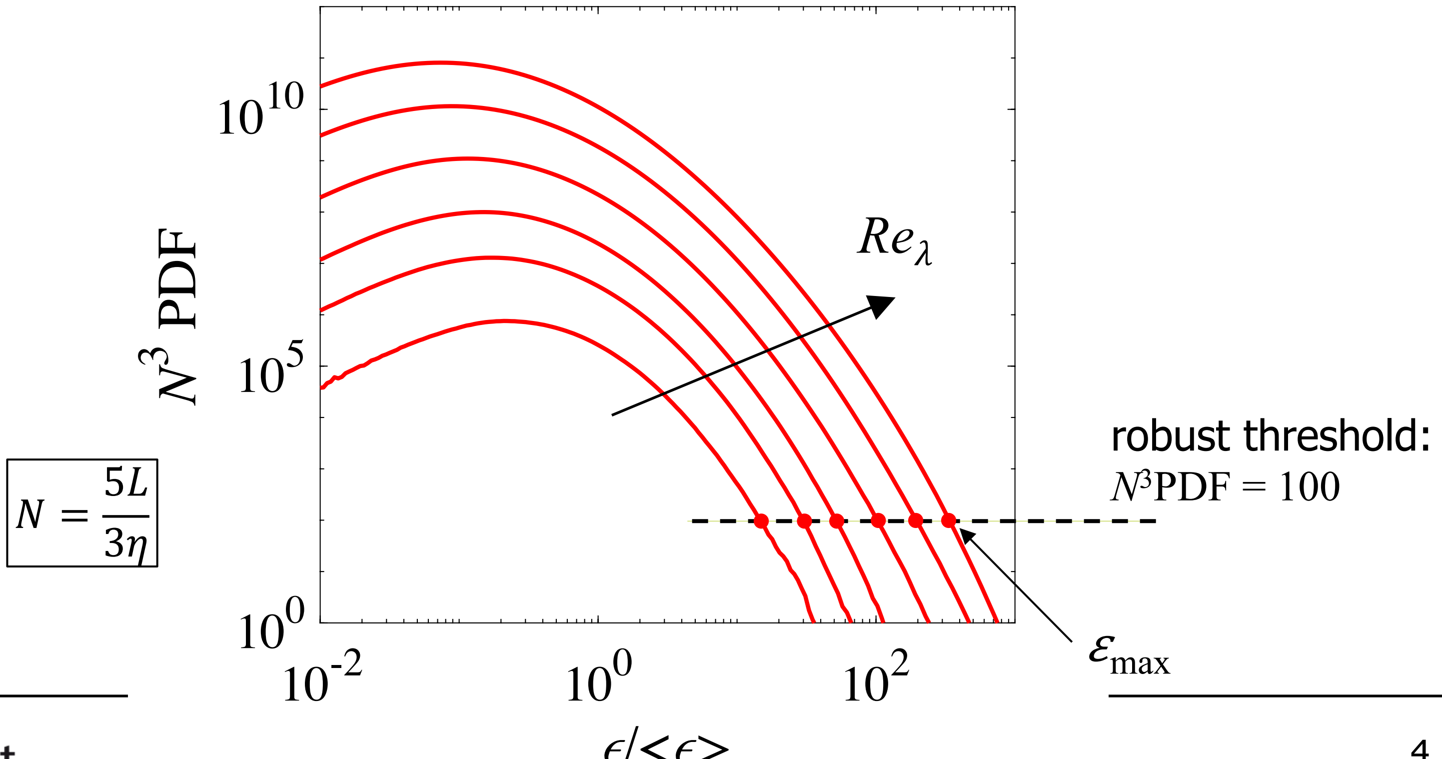
Mean dissipation-rate:

$$\langle \varepsilon \rangle = D \frac{U^3}{L}$$

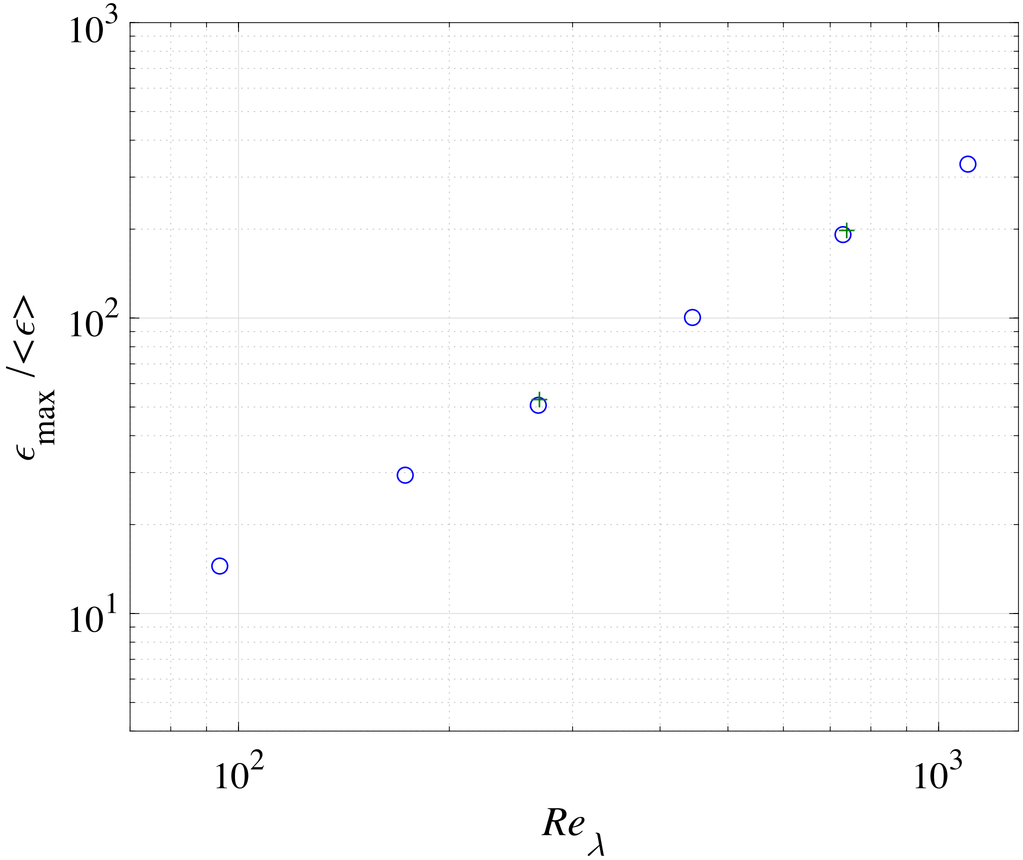
Reynolds number independent (at equilibrium conditions)

Define maximum dissipation-rate in a flow volume

- Account for the increase in the number of small-scale structures as Re_λ increases
  -> **histogram** for a $(5L)^3$ volume sampled at 3η intervals in each direction



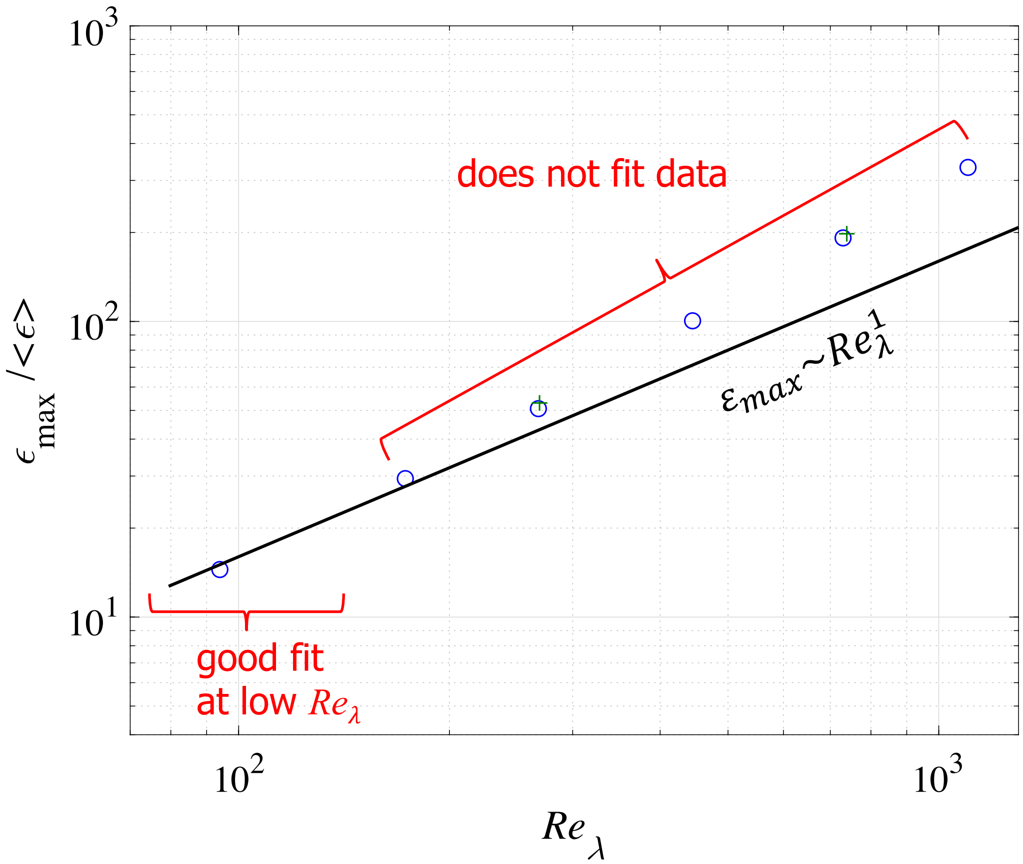
Challenge: theories and fits are inaccurate



<u>Figure uses DNS data from:</u>
Ishihara et al. 2007 JFM
Ishihara et al. 2016 Phys Rev Fluids

Some of which has been extended in terms of enhanced resolution ($k_{max}\eta=2$-4) or longer time integration

Challenge: theories and fits are inaccurate



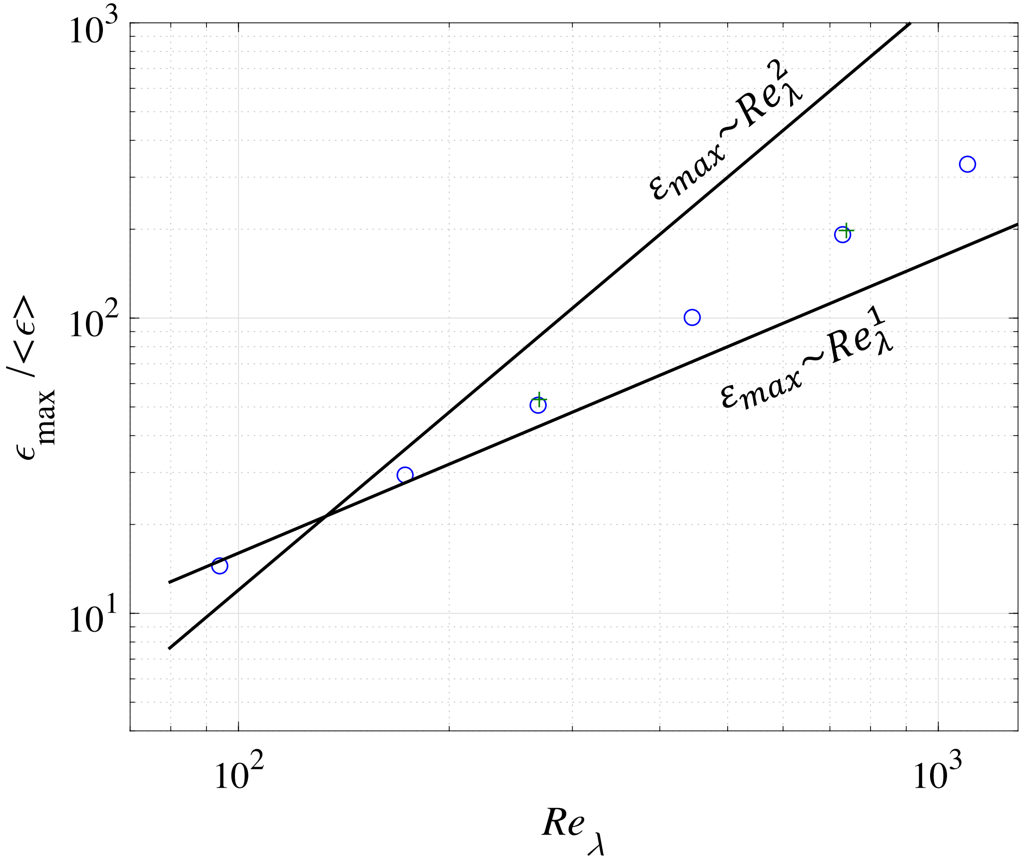
low Re_λ theory:

velocity gradients scale according to U/η

hence: $\varepsilon_{max} \sim Re_\lambda^1$

consistent with observed scaling of velocity gradients associated with intense vortices
[Jiménez et al. 1993 JFM]

Challenge: theories and fits are inaccurate



Multifractal theory:

$$\varepsilon_{max} \sim Re_\lambda^2$$

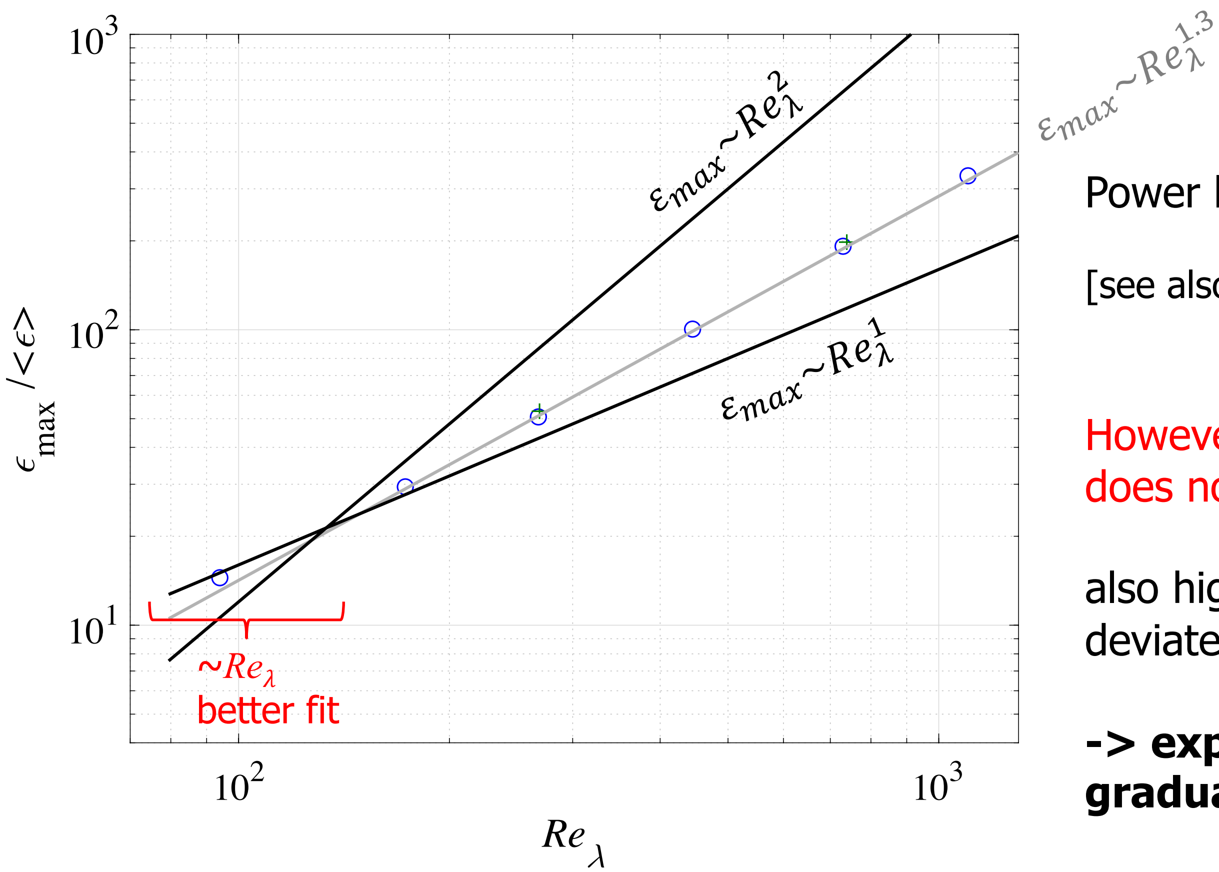
[e.g. Paladin & Vulpiani 1987, Sreenivasan & Meneveau 1988, Yakhot & Sreenivasan 2005]

Does not fit the data

Perhaps in the limit of

$Re_\lambda \to \infty$ **?**

Challenge: theories and fits are inaccurate



Power law fit

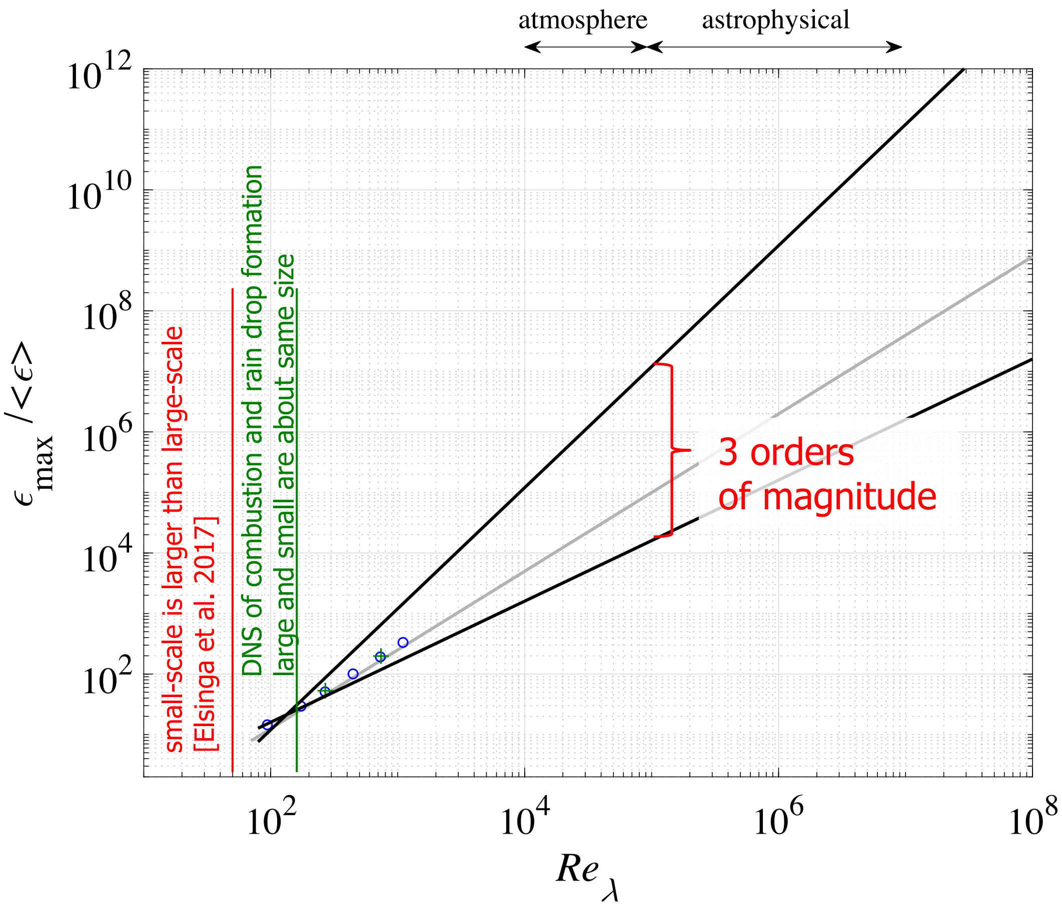
[see also Buaria et al. 2019]

However, constant power does not fit the data

also highest Re_λ seems to deviate slightly

-> exponent increases gradually with Re_λ

Challenge: extrapolation



Extrapolation is questionable since theories and power law fit do not accurately describe the data

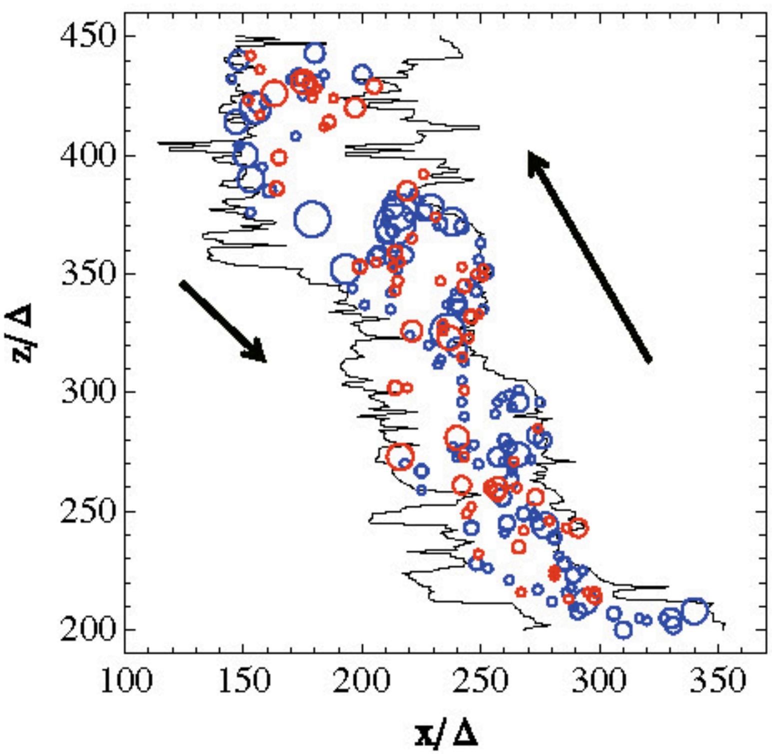
Large uncertainties at atmospheric and astrophysical conditions

Aim

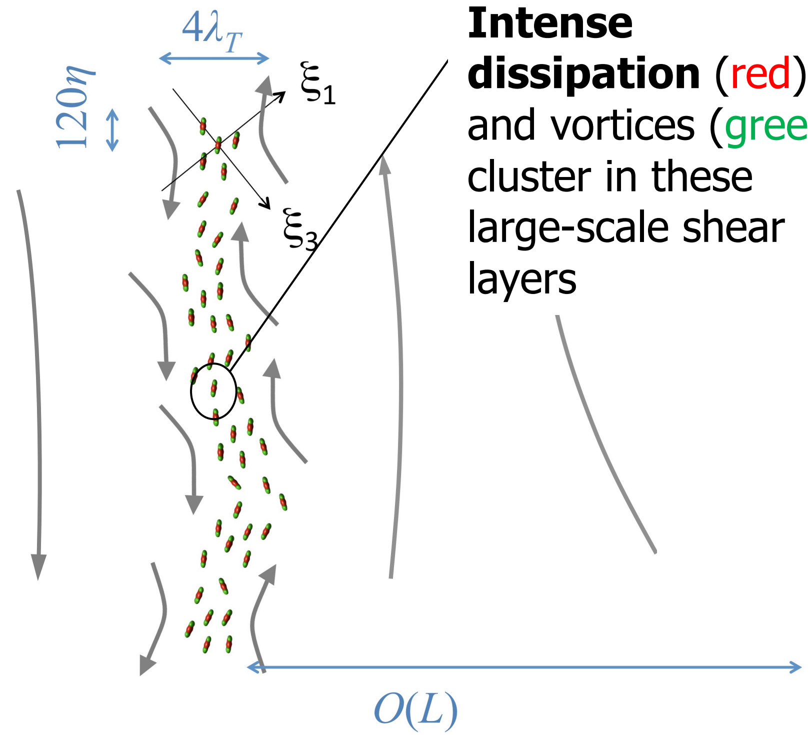
- Develop new and more accurate model for dissipation extremes
- Use knowledge of turbulent structures, large-scale shear layers in particular

Large-scale shear layers

also known as significant shear layers



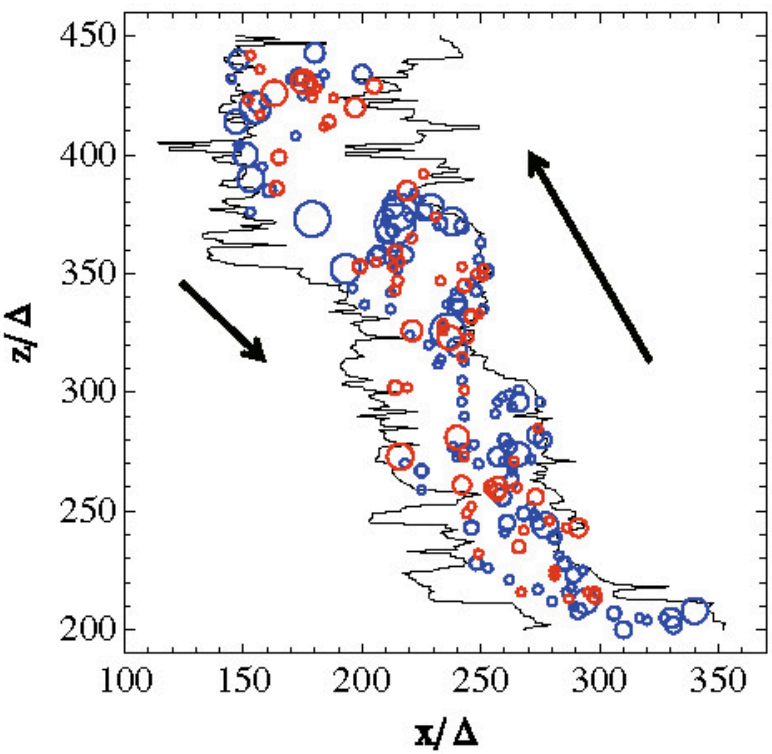
Instantaneous flow $Re_\lambda = 1100$
[Ishihara, Kaneda & Hunt 2013]

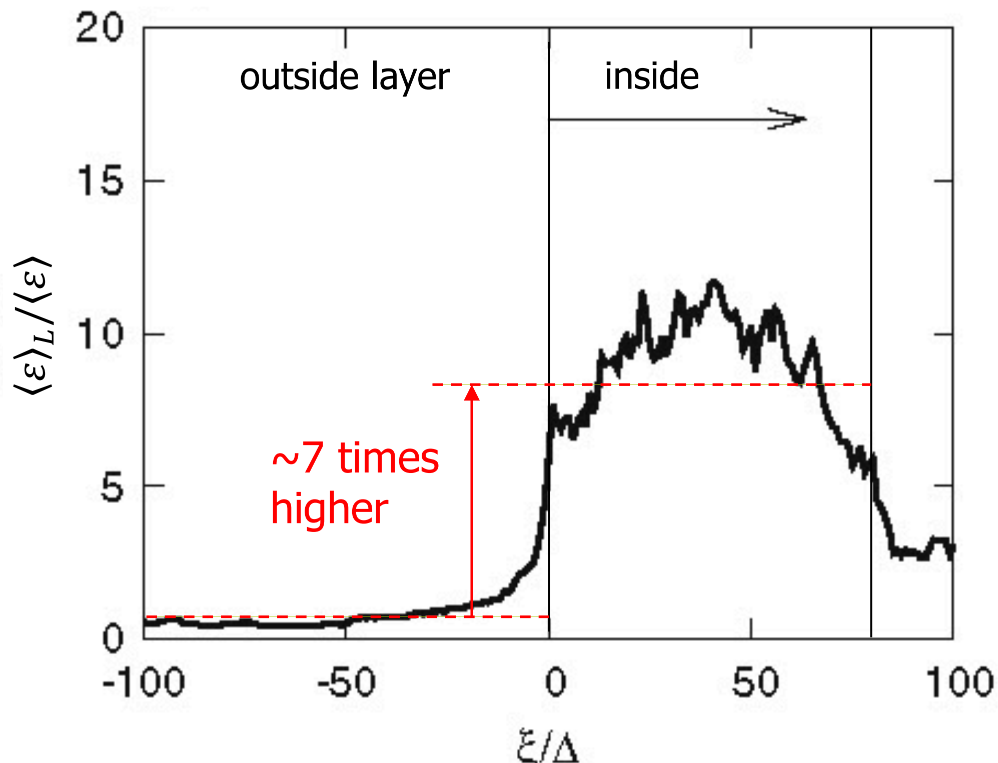


Intense dissipation (red) and vortices (green) cluster in these large-scale shear layers

Statistical evidence and Re_λ dependence
[Elsinga et al. 2017]

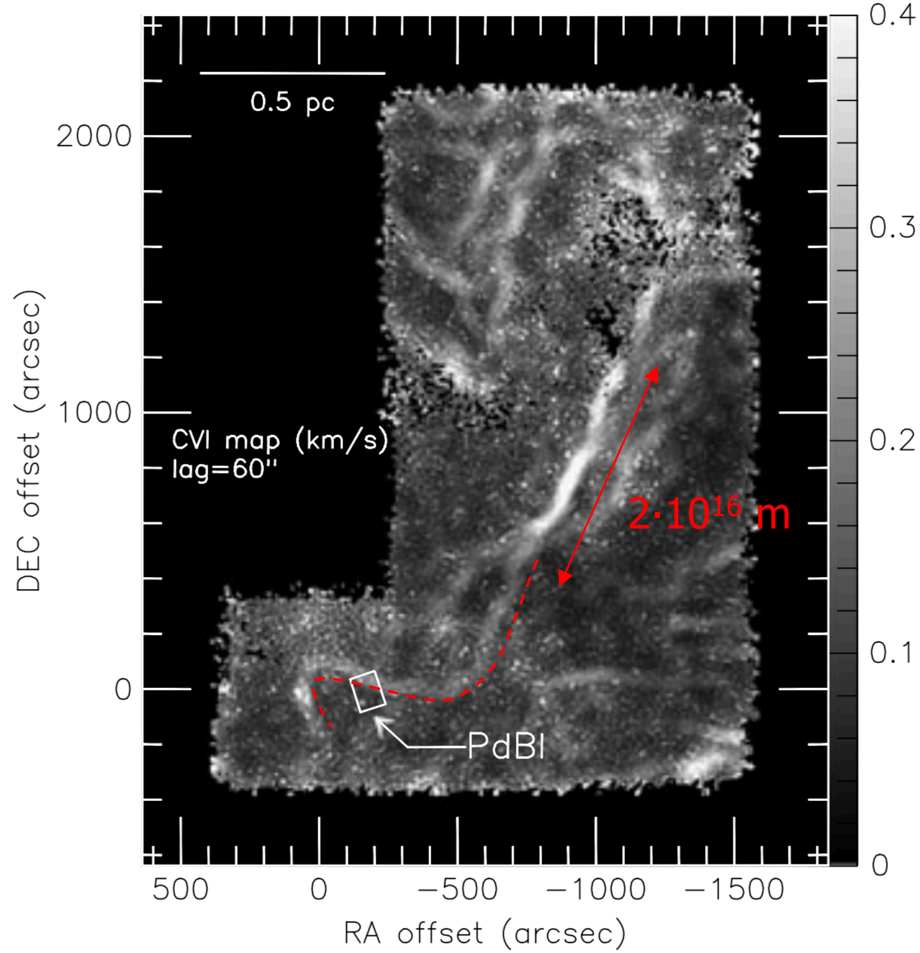
Large-scale shear layers -
elevated levels of local mean dissipation





Conditional average dissipation-rate
[Ishihara, Kaneda & Hunt 2013]

Large-scale shear layers - at very high Reynolds numbers



Map of velocity increments over a short distance (proxy for shear rate)

Polaris Flare ($Re_\lambda \sim 10^5$)
[Hennebelle & Falgarone 2012]
[Hily-Blant et al. 2008]

Modelling step 1
intermittency

Volume occupied by layers V^* relative to entire volume V:

$$\frac{V^*}{V} \propto \frac{\lambda_T L^2}{L^3} = \frac{\lambda_T}{L} \qquad => \qquad \frac{V^*}{V} = \alpha^{-1} Re_\lambda^{-1}$$

where
L – integral length scale
λ_T – Taylor scale
α – coefficient

$\alpha = 0.010$, which is consistent with $4\lambda_T$ thick layers and L wide large-scale regions

Modelling step 2
local average dissipation rate

Define:

- background dissipation rate (constant over entire volume),

$$\varepsilon_{bg} = b\langle\varepsilon\rangle$$

- dissipation rate averaged over the layer, ε^*

Balance:

$$\langle\varepsilon\rangle V = \left(\varepsilon^* - \varepsilon_{bg}\right)V^* + \varepsilon_{bg}V$$

total = excess in layers + background

$$=> \quad \varepsilon^* = \langle\varepsilon\rangle[b + (1-b)\alpha Re_\lambda]$$

coefficient $b = 0.67$, which is consistent with $\varepsilon^*/\varepsilon_{bg} = 6.4$ at $Re_\lambda = 1100$ [Ishihara, Kaneda & Hunt 2013]

Modelling step 3
layer turbulence & substructures

The significant shear layers are turbulent, which is characterized by a local Reynolds number, ${Re_\lambda}^*$

Define:

- local integral scale: $L^* = \lambda_T$ Such that $4L*$ fit across the layer
- local Kolmogorov scale: $\eta^* = \eta \left(\frac{\langle \varepsilon \rangle}{\varepsilon^*} \right)^{1/4}$

This range of scales defines a ${Re_\lambda}^*$:

$${Re_\lambda}^* = \left[15^{3/4} D^{-1} \frac{L^*}{\eta^*} \right]^{2/3}$$

where
η – global Kolmogorov length scale
D – normalized mean dissipation rate

Modelling step 3

layer turbulence & substructures

The significant shear layers appear fully developed
when $Re_\lambda = 250$ [Elsinga et al. 2017 JFM]
or underdeveloped when $Re_\lambda = 150$ [Elsinga & Marusic 2010 JFM]

Similarly, sublayers develop with significant shear layer
when $Re_\lambda{}^* = 150$ (corresponding to $Re_\lambda = 1560$)

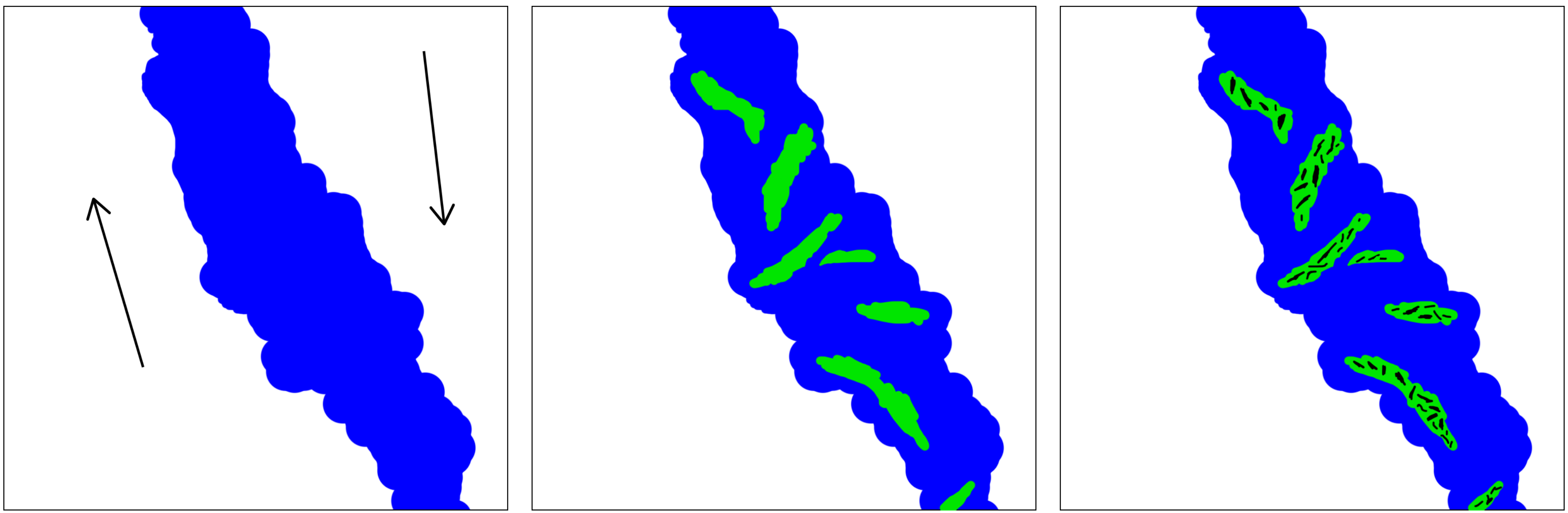
… and when $Re_\lambda{}^* = 1560$ (corresponding to $Re_\lambda = 1.8 \cdot 10^5$)
sub-sublayers develop within the sublayers

… and so on …

[note: some evidence of sublayers is provided by observations in molecular clouds, see Falgarone et al. 2009 A&A]

Modelling step 3
layer turbulence & substructures

significant shear layers $Re_\lambda > 150$ | **sublayers** $Re_\lambda > 1560$ | **sub-sublayers** $Re_\lambda > 1.8 \cdot 10^5$



Modelling step 3
layer turbulence & substructures

The conditions in the **sublayers follow the same relations** as developed for the significant shear layers

simply **use the local conditions**
replacing Re_λ with ${Re_\lambda}^*$, $<\varepsilon>$ with ε^*, etc…

For example,

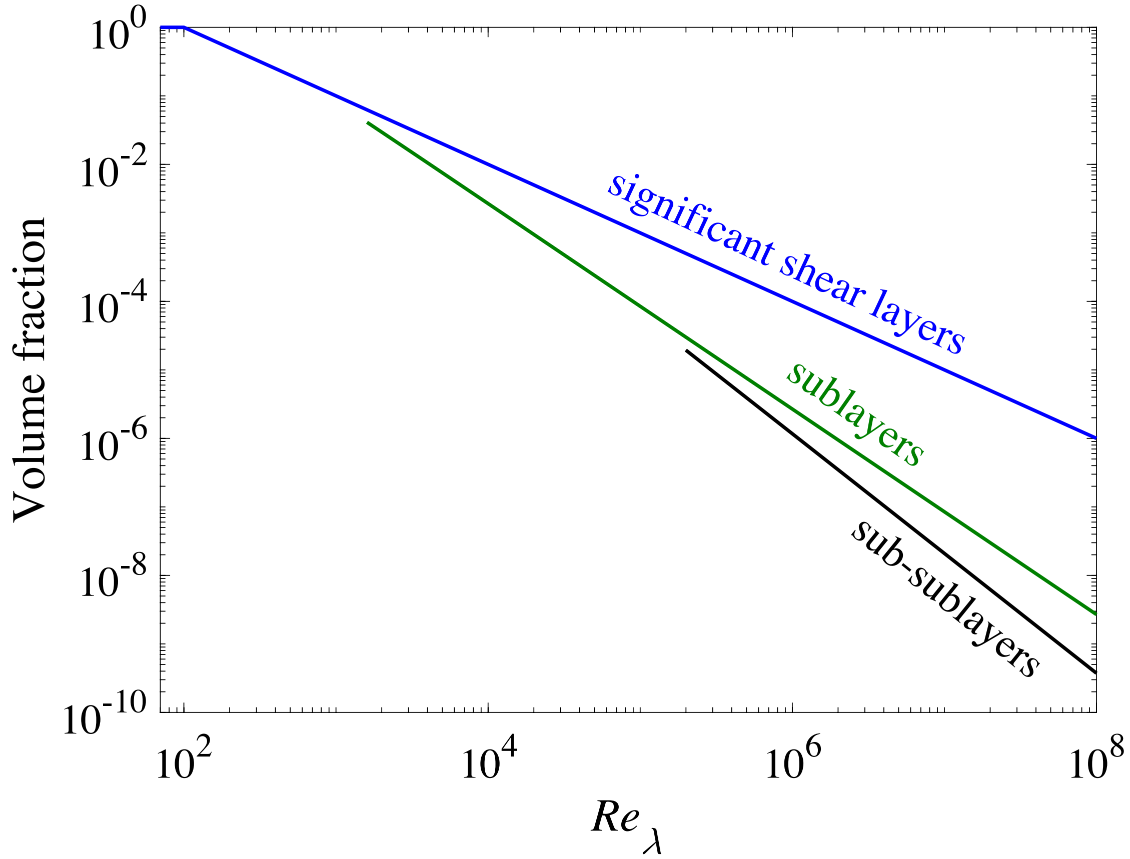
$$\varepsilon^* = \langle\varepsilon\rangle[b + (1 - b)\alpha Re_\lambda]$$

becomes

$$\varepsilon^*_{sublayer} = \varepsilon^*[b + (1 - b)\alpha {Re_\lambda}^*]$$

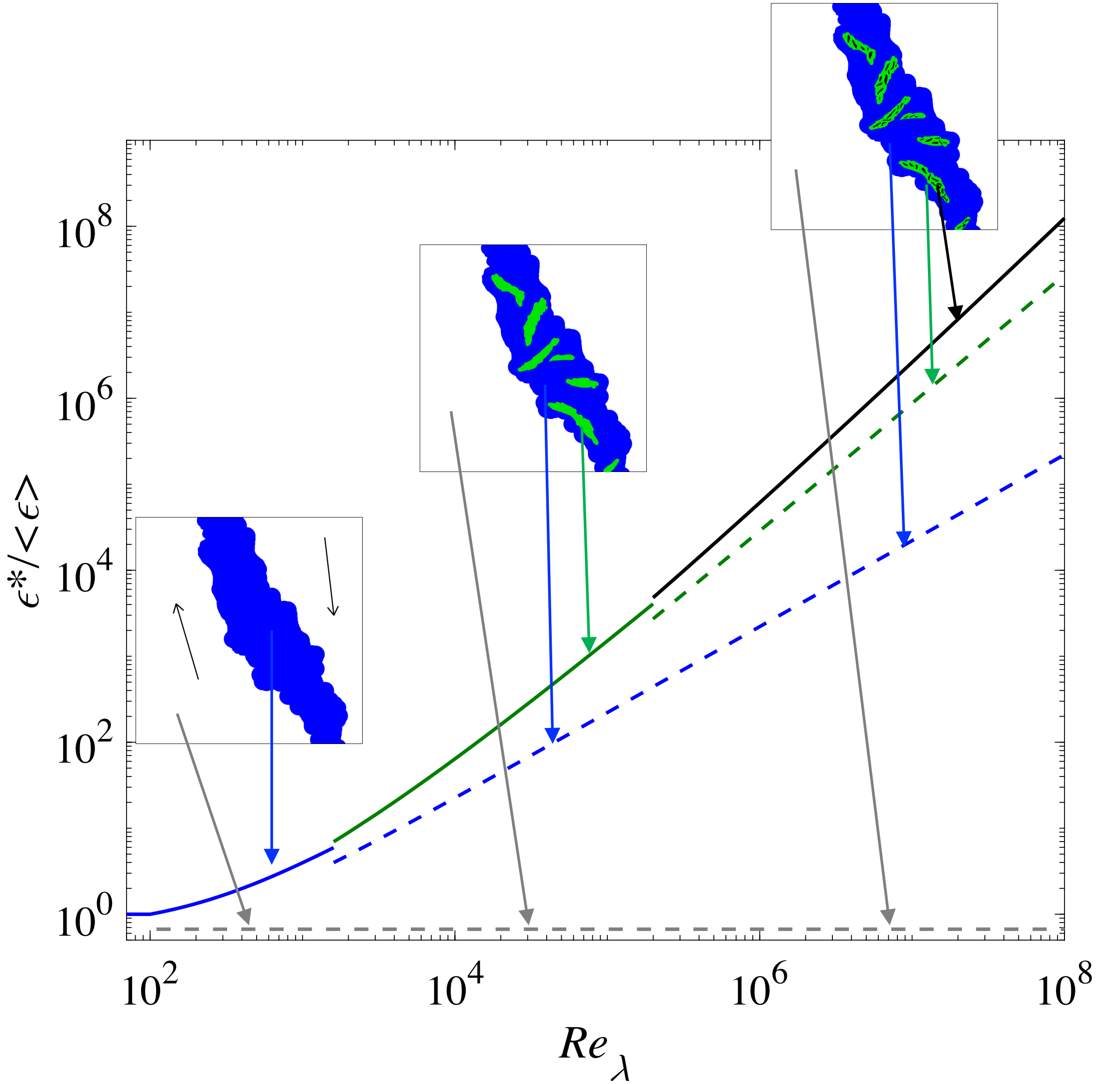
Results

intermittency



Results

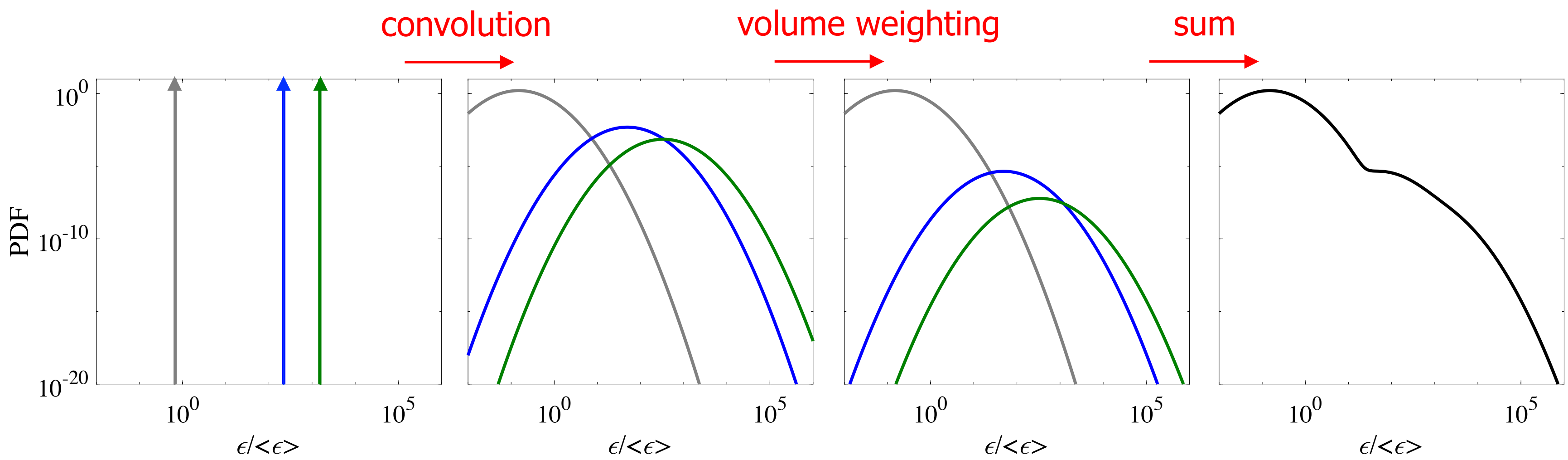
local average dissipation rate



Modelling step 4
convolve with lognormal distribution & obtain overall dissipation-rate PDF

The local PDF of the dissipation-rate for each flow region (background, significant shear layer, sublayer, …) is given by a lognormal distribution centered on the local average dissipation-rate

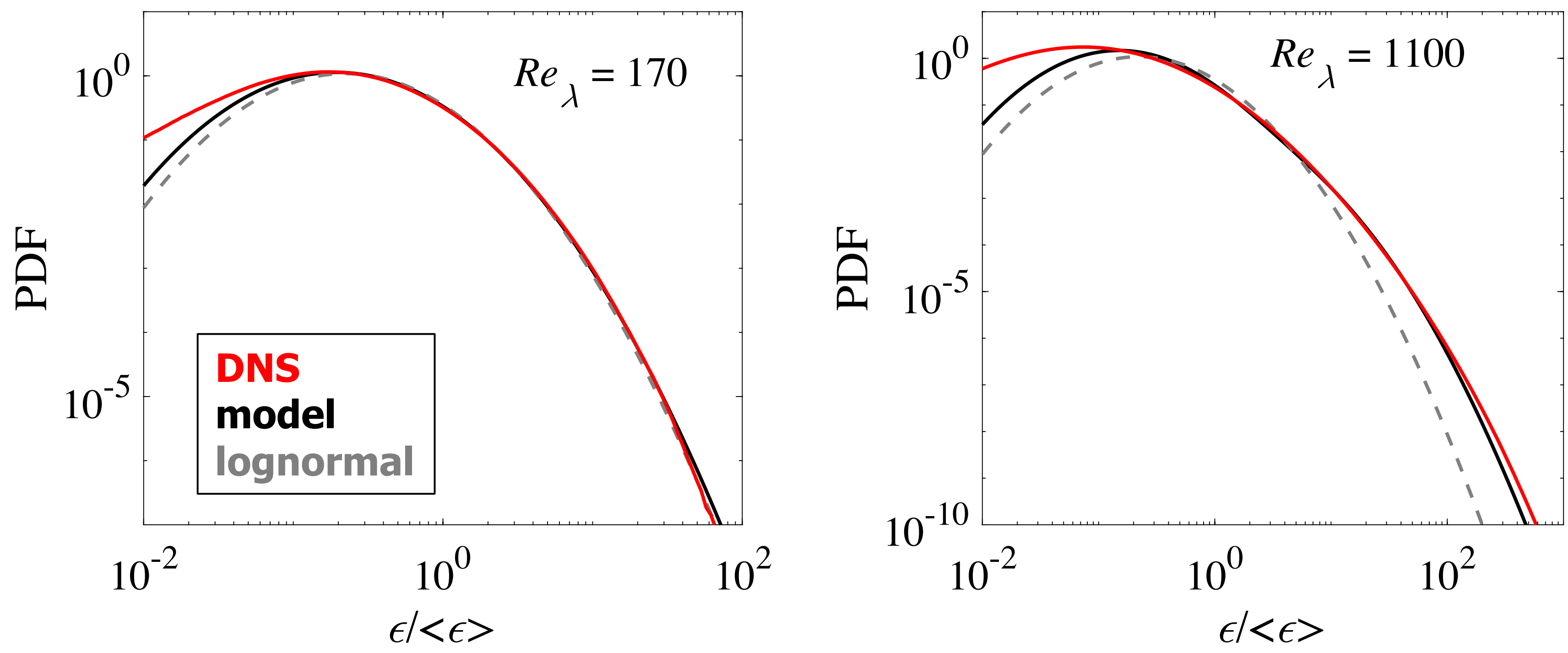
The overall PDF is the volume weighted average of these local PDFs.



Example shows $Re_\lambda = 10^5$

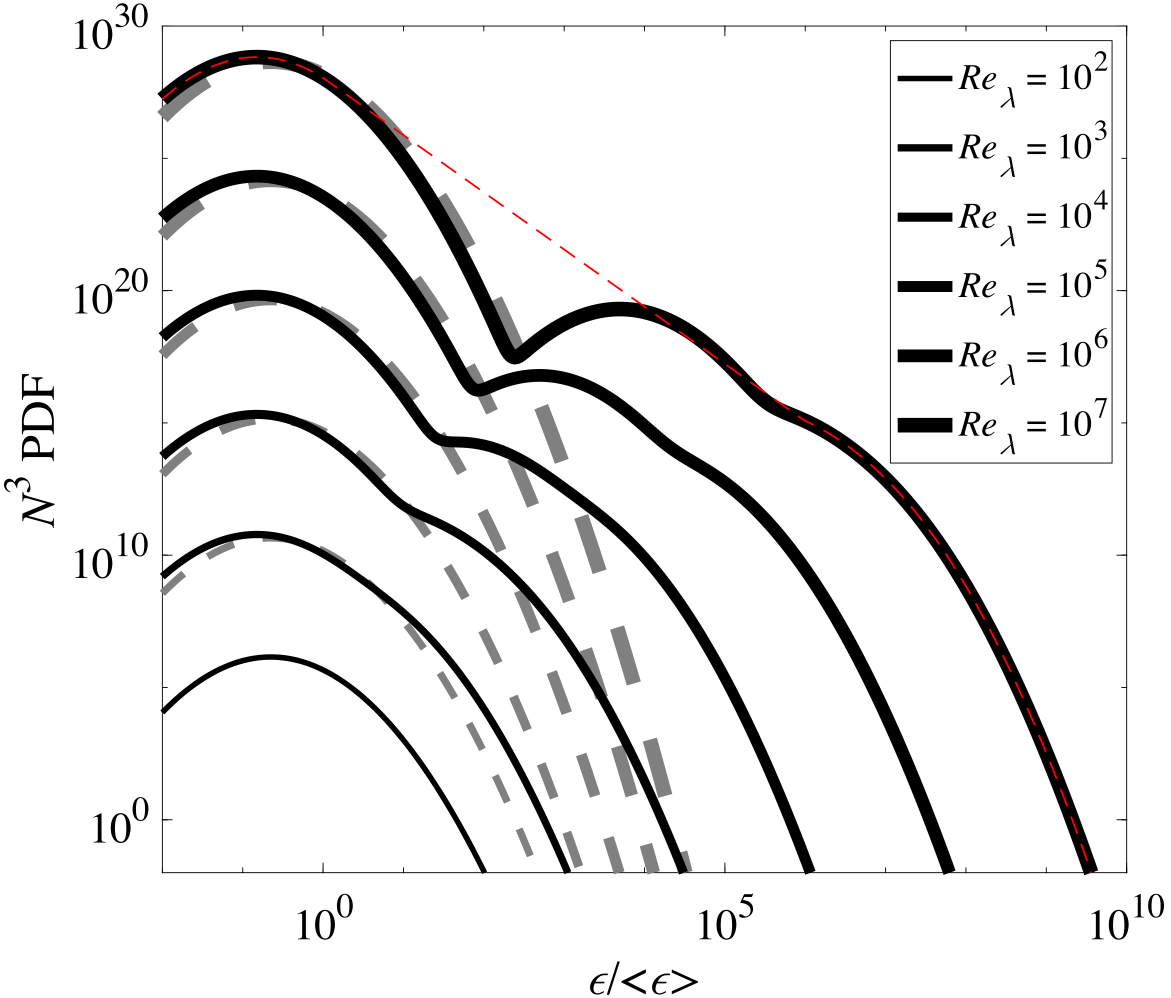
Results

overall dissipation-rate PDF



Results

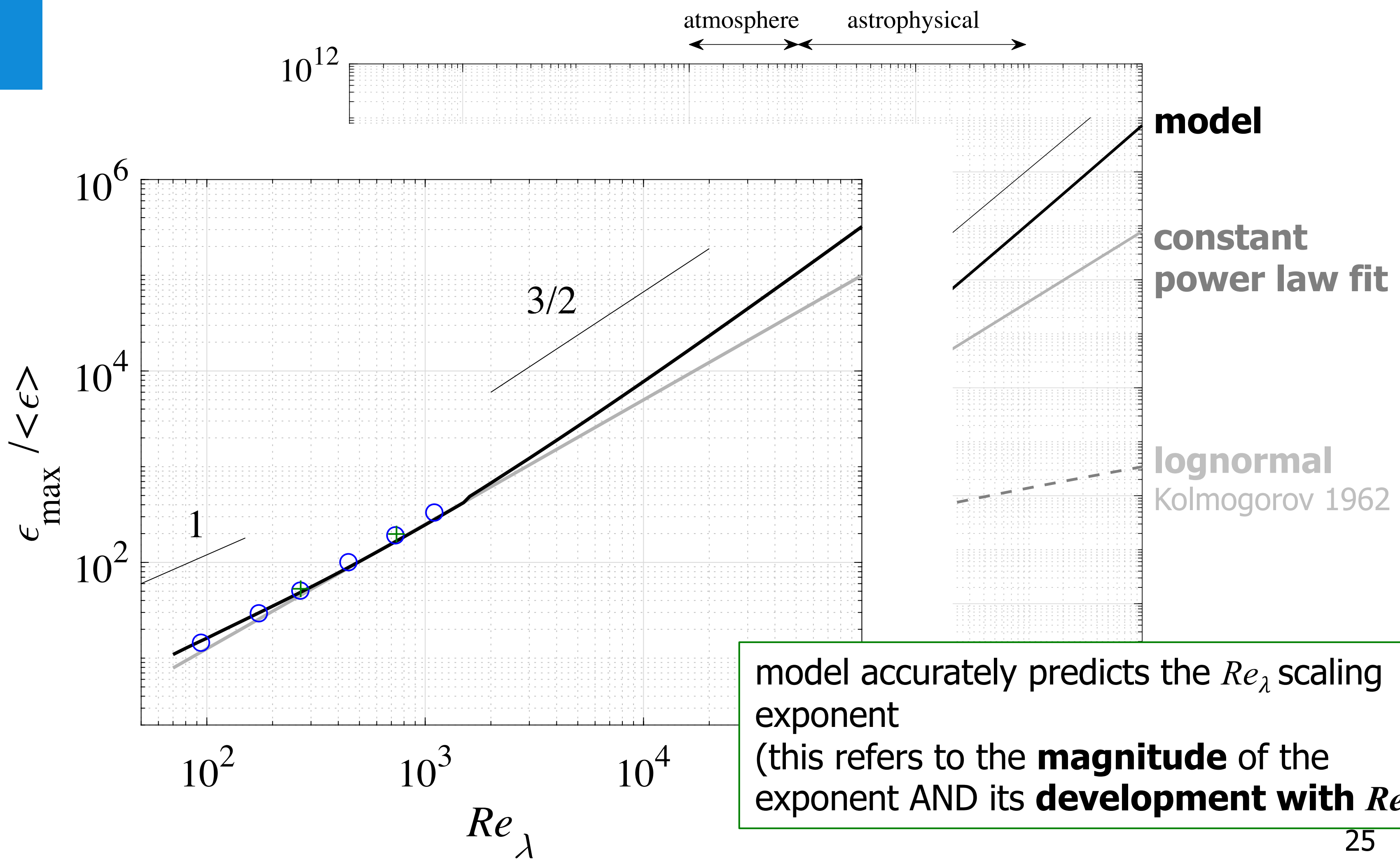
Reynolds number effect



model
lognormal

Results

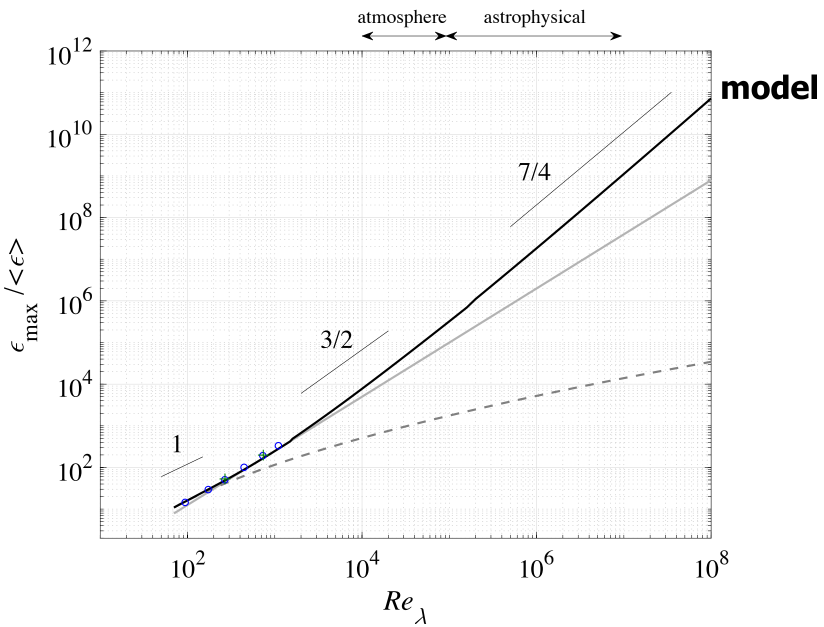
Maximum dissipation-rate



model accurately predicts the Re_λ scaling exponent
(this refers to the **magnitude** of the exponent AND its **development with Re_λ**)

Results

Infinite Reynolds number limit



In the limit of $Re_\lambda \rightarrow \infty$

additional layered substructure develops and the model ultimately **approaches Multifractal theory**:

$$\varepsilon_{max} \sim Re_\lambda^2$$

However,

$\varepsilon_{max} \sim Re_\lambda^{1.95}$ is reached only when $Re_\lambda \approx 10^{40}$

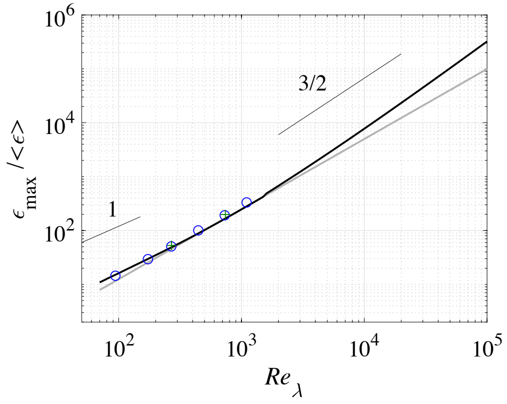
Consequently, finite value Re_λ remains important in any real application

Conclusions

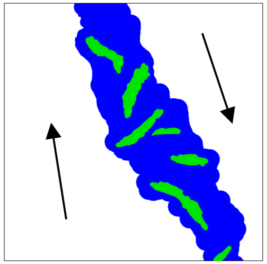
Significant shear layers are intrinsic to explanation & quantification of extreme dissipation

Model needs to incorporate relation between large and small scale

AND smallest scale is not η



Our model accurately predicts the **magnitude** of Re_λ scaling exponent AND its **development** with Re_λ over the range where data is available



Predict the development of sublayers and sub-sublayers at (very) high Reynolds numbers

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